SMSTC Prospectus 2023-24

Welcome to the Scottish Mathematical Sciences Training Centre. This document gives a brief overview for students and their supervisors of what SMSTC offers. We hope that it will be useful to you in planning the first few months of your PhD – and beyond!

SMSTC modules fall into two categories. **Core modules** are offered every year, and are formally assessed. **Supplementary modules** may change from year to year, and they may or may not be assessed. Supplementary modules are not necessarily more advanced than core modules, but often deal with a special topic of current research interest. All SMSTC students are welcome to attend both core and supplementary modules.

What we offer is designed to be flexible. SMSTC allows students to enrol in as many or as few modules as they like, in the first year or in later years of their PhDs; and to enrol in any module without taking the assessments. (However, many departments do have more specific requirements, so students are advised to discuss their choice of modules with their supervisors.) Students often gain most from taking a mixture of modules that relate directly to their research interests and modules that will **broaden their mathematical knowledge** in other areas – the latter may be more challenging, but also rewarding!

How, what, and when

SMSTC lectures are delivered by **video conference using Zoom**; departments provide local tutorial support, which is not co-ordinated by SMSTC. Lecture notes and other material may be found on our **website**, www.smstc.ac.uk. To access this material, students will need to **register** with SMSTC and then **enrol** for each module; instructions can be found on the front page of the site.

The SMSTC year begins with the **opening symposium**, which will be held on 4th – 5th October 2023 in Perth. The symposium will include overviews of the modules, as well as sessions dealing with practical topics (such as how to get a PhD) and opportunities to meet your fellow students. We intend to follow this with further professional development events for PhD students later in the year, please look out for further details.

Semester 1 runs from Monday 9th October 2023 to Friday 15th December 2023, and **Semester 2** runs from Monday 15th January 2024 to Friday 22nd March 2024.

Each core module will be assessed by one or more **assignments**. The deadline for the final assignment is typically two or three weeks after the end of the semester, and we aim to return grades a few weeks after that. Letters will be issued to students in late May confirming the modules that they have taken and the grades they have received.

More information about the structure of SMSTC and the **content of the modules** is available on the website. Please see especially the SMSTC Information for Students page:

https://www.smstc.ac.uk/information/students

Modules available in 2023-24

Core modules

The sixteen core modules are organised on four themes: Analysis [ANA]; Applications of Mathematics [AOM]; Probability and Statistics [PAS]; Structure and Symmetry [SAS]. The lectures for the core modules take place in the afternoons, Monday through Thursday, throughout each semester (see the timetable on the SMSTC website). The titles of the core modules and their themes are as follows:

Semester 1

Representation Theory [SAS]
Dynamical Systems and Conservation Laws [ANA]
Asymptotic and Analytical Methods [AOM]
Algebraic Topology [SAS]
Continuum Mechanics [AOM]
Foundations of Probability [PAS]
Measure and Integration [ANA]
Regression and Simulation Methods [PAS]

Semester 2

Algebraic Geometry [SAS]
Elliptic and Parabolic PDEs [ANA]
Numerical Methods [AOM]
Differential Topology [SAS]
Mathematical Biology and Physiology [AOM]
Stochastic Processes [PAS]
Functional Analysis [ANA]
Modern Regression and Bayesian Methods [PAS]

Supplementary modules

In 2023-24 we will offer a selection of supplementary modules. Lectures will take place in the mornings, Monday through Thursday, throughout each semester. However, some modules may not use the full ten weeks. Details will be communicated to you by the module leader.

Note that supplementary modules are not necessarily assessed, so if you need to take a module for credit then you should check the module page (and ask the lecturer if necessary).

Semester 1

Geometry of Gauge Fields
Riemann Surfaces

Semester 2

Advances in Numerical Methods for Hyperbolic and Kinetic Equations
Calculus of Variations
Finite Element Methods for PDEs
History of Mathematics
Homological Algebra
Hopf Algebras
Introduction to Geometric Group Theory

Which modules are suitable for me?

Every student taking SMSTC will have a different academic background and interests, so modules are not labelled as “introductory” or “advanced”. The following list of expected prior knowledge should give you an idea of where each module starts relative to what you've already studied. Before making any choices, though, you should also look at the syllabus (see the module page on the website) and if possible attend the symposium where you will be able to speak to the module leader or theme head.

Core modules: expected prior knowledge

Algebraic Geometry. The notion of a module and related concepts; basics on Noetherian and Artinian modules; some commutative algebra, in particular the notion of a principal ideal domain.

Algebraic Topology. A working knowledge of metric and topological spaces; linear algebra (vector spaces, linear maps and quotient vector spaces); group theory (groups and group actions).

Asymptotic and Analytical Methods. Basic ODEs (first-order separable and first- and second-order linear equations); single- and multivariable calculus; Taylor’s theorem; linear algebra; contour integration including Cauchy's theorem.

Continuum Mechanics. Introductory courses on ODEs, PDEs, vector calculus and basic linear algebra.

Differential Topology. Multivariable calculus, Point-Set Topology, and a standard first course in Differential Geometry, for example on curves and surfaces. We recommend other interested students to get in touch with the lecturer(s). Having taken the SMSTC algebraic topology course, or an equivalent, is recommended. Some familiarity with Lie groups and Lie algebras would be useful, but is not essential.

Dynamical Systems and Conservation Laws. Undergraduate-level ODEs, single- and multivariable real analysis, and linear algebra.

Elliptic and Parabolic PDEs. Undergraduate-level ODEs, single- and multivariable real analysis, and linear algebra.

Foundations of Probability. Elements of mathematical analysis, linear algebra and combinatorics at undergraduate level.

Functional Analysis. Undergraduate analysis: sequences, series, pointwise and uniform convergence. Metric space topology: at least in R^d, continuity of functions, open, closed and compact sets. Countable sets. Some of the examples draw upon the measure theory from “Measure and Integration”.

Mathematical Biology and Physiology. Undergraduate-level knowledge of ODEs, PDEs, vector calculus and basic linear algebra.

Measure and Integration. Undergraduate analysis: sequences, series, pointwise and uniform convergence. Metric space topology: at least in R^d, continuity of functions, open, closed and compact sets. Countable sets.

Modern Regression and Bayesian Methods. SMSTC “Regression and Simulation Methods” or equivalent.

Numerical Methods. Basic ODEs (in particular first order separable and first- and second-order linear equations); single- and multivariable calculus; Taylor’s theorem; and linear algebra. This module

involves coding; prior knowledge of Matlab or Octave would be helpful, and students who do not have prior experience of a programming language should be aware that they will need to learn one.

Regression and Simulation Methods. Basic concepts in: probability (elementary probability distributions); statistics (ideas of estimation, confidence intervals, hypothesis tests); calculus. The level required in these areas would usually be provided in a first undergraduate course.

Representation Theory. Basic linear algebra; definitions and examples of groups, rings, fields; basic algebra concepts such as homomorphisms; basic notions of group theory – permutations, symmetric groups, Lagrange's theorem, normal subgroups and factor groups.

Stochastic Processes. Elements of mathematical analysis, linear algebra and combinatorics at undergraduate level. Probability theory, either at undergraduate level or from SMSTC “Foundations of Probability”.

Supplementary modules: expected prior knowledge

Advances in Numerical Methods for Hyperbolic and Kinetic Equations. A UG level or advanced UG level course on numerical analysis including methods for ODEs, and a graduate level or advanced UG level course on PDEs with some notions on the heat and transport equations.

Calculus of Variations. Some knowledge of Sobolev spaces and elementary functional analysis an advantage.

Finite Element Methods for PDEs. A graduate level or advanced UG level course on PDEs and/or Calculus of Variations, hopefully involving some function spaces and the four basic problems of mathematical physics (Poisson, heat, wave, transport PDE problems).

History of Mathematics. Calculus, algebra, and analysis at the level of undergraduate understanding typically presumed of a postgraduate student in mathematical fields, but the discussion will be historical rather than mathematical so don't worry if you are not completely confident in all the details.

Geometry of Gauge Fields. Students taking this course should have some background in geometry and topology (definition of a manifold) and group theory, but no knowledge of Lie groups or gauge theory will be assumed.

Homological Algebra. Working knowledge of groups, rings and modules. Some familiarity with Algebraic Topology (at the level of the SMSTC course) would be useful.

Hopf Algebras. As background, basic knowledge of rings, modules and representation theory of groups would be useful as well some familiarity with category theory and commutative diagrams.

Introduction to Geometric Group Theory. This module will be accessible to anyone having taken a standard abstract algebra course at undergraduate level. A familiarity with algebraic topology or differential geometry (as in SMSTC “Algebraic Topology” and “Differential Topology”) is not required, but students with this background will be able to study additional examples connecting to their own research.

Riemann Surfaces. A knowledge of complex analysis in one variable and an appreciation for the basic ideas of topology and analysis.